

Space, shape and measures diagnostic test

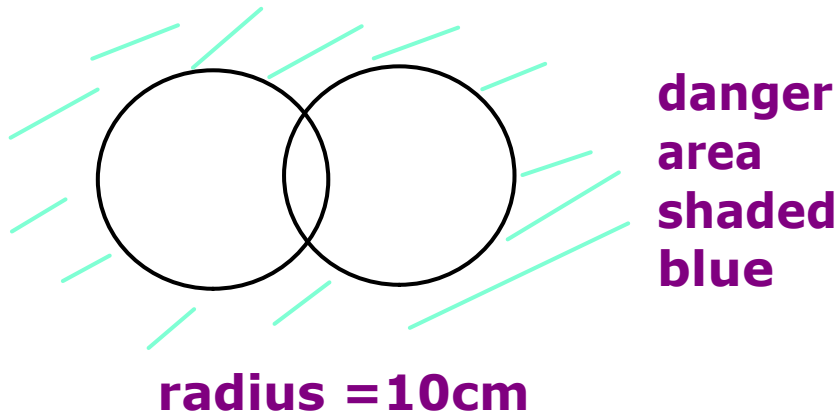
1. Name 4 polygons which tessellate. **Triangles, squares, rectangles, parallelograms...**
2. How big are the external angles of a regular octagon? $360/8 = 45^\circ$

3. a) A triangle ABC with A at (1,4); B at (1, 6) and C at (3, 5) is reflected in the line $y=x$ to form $A'B'C'$. Find the co-ordinates of C' .
b) ABC is enlarged scale factor 2, centre (1,0) to form $A''B''C''$. Find the co-ordinates of A'' .
c) If the base of ABC is 3.5cm work out the height.
d) Find the area of ABC

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4. a) A car journey takes 2 hours 37 mins to cover 150 miles. What is the average speed to 3 sig fig?
Remember to change minutes to decimals
 $sp = \text{dist}/\text{time} = 150 / 2.61666 = 57.3248 = 57.3\text{mph}$
b) If it had travelled at 56 miles per hour how long would the journey have taken to the nearest minute?
 $sp = \text{dis}/\text{time}$ so $56 = 150/t$ so $56t = 150$ or $t = 150/56 = 2.67857\text{hrs} = 2 \text{ hrs } 41 \text{ mins}$

5. Two allied boats have a firing range of 200metres. They are 180 metres apart.
Draw a scale diagram using 1:2000 to show the danger area around them.



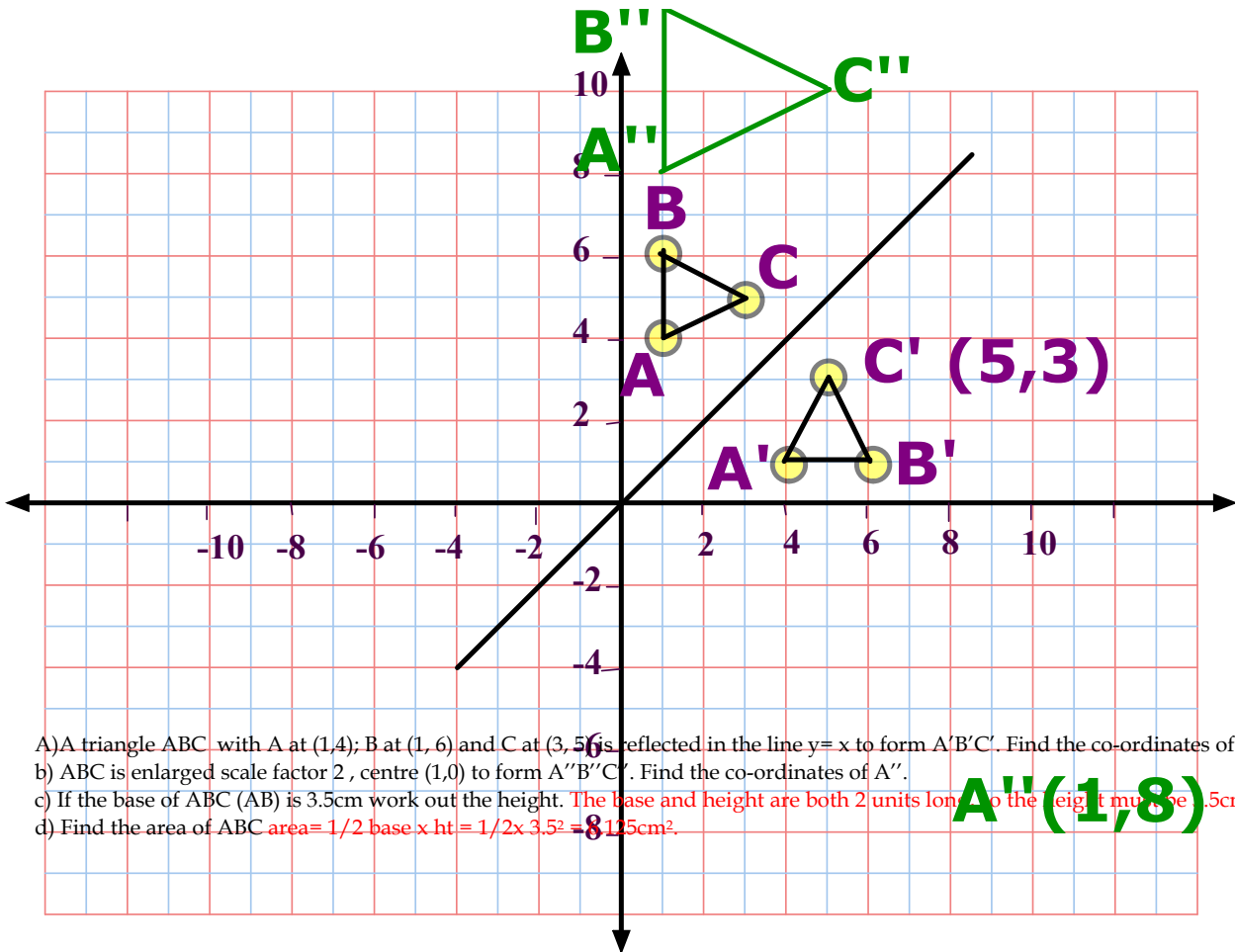
6. A sparrow perched itself on the end of a minute hand of a church clock. The minute hand was 33cm long, and the sparrow sat there for 10 mins. How far did it travel to the nearest mm?

10 mins is 60° (one sixth of the circumference)

$$\begin{aligned} \text{length of an arc} &= \text{angle}/360 \times \pi \times d \\ &= 60/360 \times \pi \times 66 \\ &= 11\pi \\ &= 34.6\text{cm} \end{aligned}$$

7. Which of these could describe area:

πabc , $\pi^2 h$, abc , $a^2 b$, **ab only**
(vol); (length); (vol), (vol), area



- A) A triangle ABC with A at (1,4); B at (1, 6) and C at (3, 5) is reflected in the line $y=x$ to form $A'B'C'$. Find the co-ordinates of C' .
- b) ABC is enlarged scale factor 2, centre (1,0) to form $A''B''C''$. Find the co-ordinates of A'' .
- c) If the base of ABC (AB) is 3.5cm work out the height. **The base and height are both 2 units long so the height must be 3.5cm.**
- d) Find the area of ABC **area = $\frac{1}{2}$ base x ht = $\frac{1}{2} \times 3.5^2 = 8.125\text{cm}^2$.**

A''(1,8)

8. Two similar cartons are used for selling milk. The smaller has a base length of 8cm, and the larger has a base length of 20 cm. what is the ratio of the lengths
 b) If the smaller one has a height of 12 cm, what is the height of the larger one?
 c) What will the ratio of their volumes be (in its simplest form)?

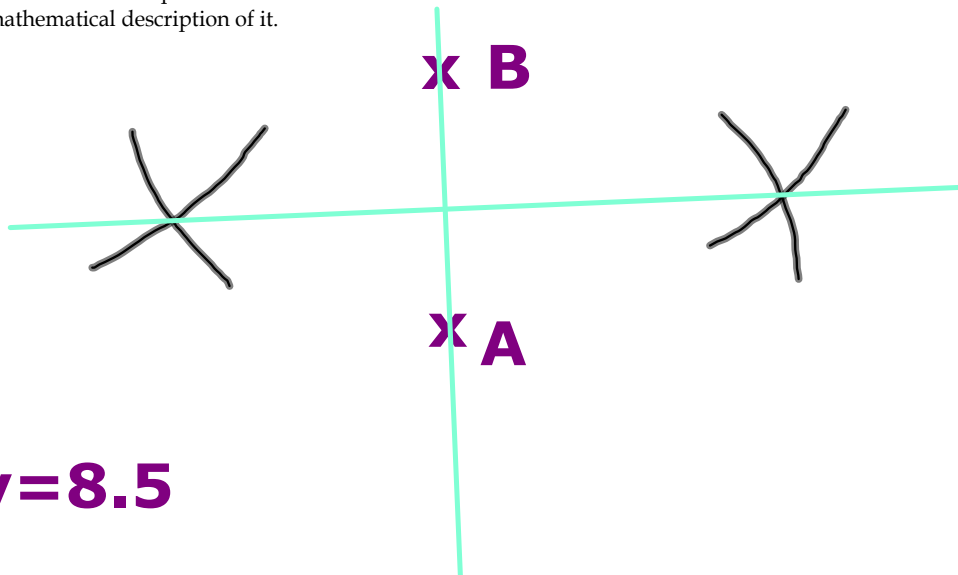
a) 8:20 or 2:5

b) $2:5 = 12 : 30$

so larger one has ht 30cm

c) $2^3 : 5^3 = 8:125$

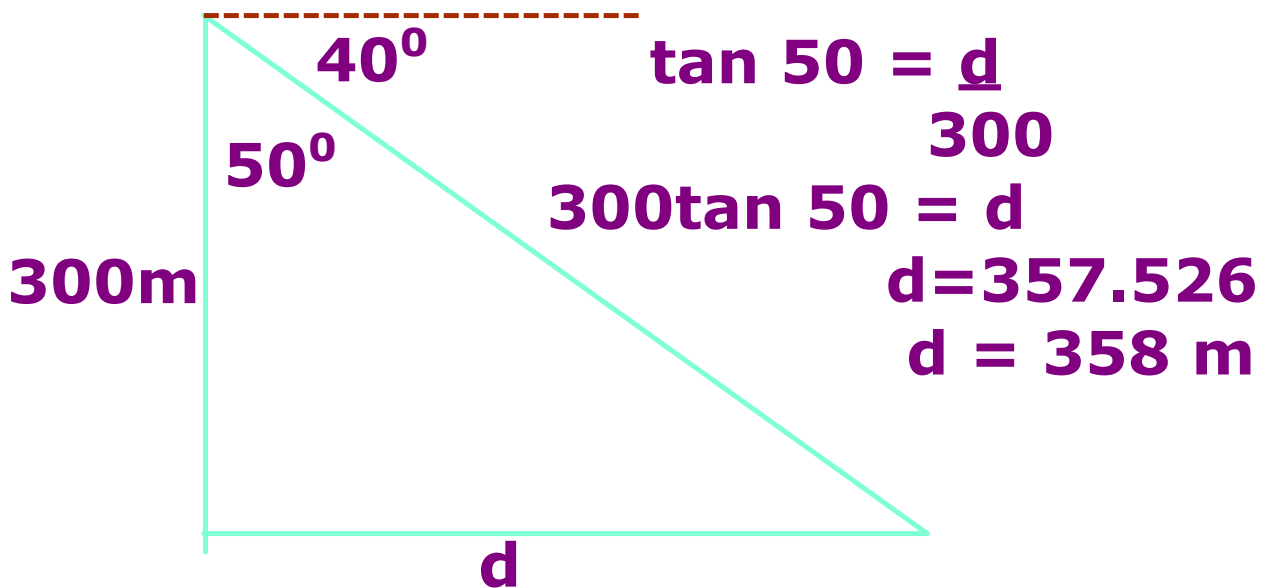
9. Plot A at (3,7) and B at (3, 10) .
 Construct the set of points that are all equidistant from A and B.
 Name this line with an equation.
 Give a mathematical description of it.

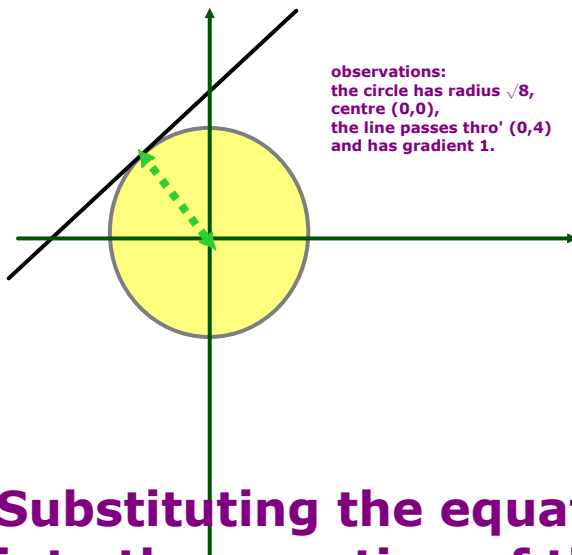


$y=8.5$

the perpendicular bisector

10. A woman standing at the top of the Eiffel tower looks down at Paris below through a pair of binoculars. If the binoculars point down with an angle of depression 40 degrees and the Eiffel tower is 300m high how far (to the nearest metre) from the base of the tower can the woman see? (Ignore her height)





Substituting the equation of the line into the equation of the circle:

$$\text{a) } x^2 + (x+4)^2 = 8$$

$$x^2 + (x^2 + 8x + 16) = 8$$

$$2x^2 + 8x + 8 = 0$$

$$x^2 + 4x + 4 = 0$$

$$(x+2)(x+2) = 0$$

$x = -2$ (twice) therefore a tangent!

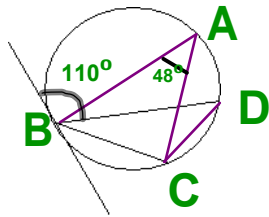
b) The tangent must have gradient 1 (since the other has gradient 1 and they are parallel).

The circle has centre $(0,0)$ so the other tangent must pass through -4 on the x axis (equidistant from $(0,0)$).

Equation is $y = x - 4$

You could substitute in again, or now use the fact it must be symmetrical to say the point of intersection must be at

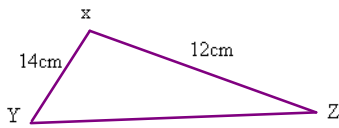
$$\mathbf{x = 2, y = -2}$$



- a) Find angle BDC
 - b) Angle DBC
- Give reasons for your answers.

Angle BDC = 48°
Angles at the circumference are equal.

Angle DCB = 110 (Alternate segment theorem)
Angle DBC = 22° (Angles in a triangle total 180°)



$$\text{Area} = \frac{1}{2} ab \sin C$$

$$50 = \frac{1}{2} \times 14 \times 12 \times \sin C$$

$$\frac{100}{168} = \sin C$$

$$C = \sin^{-1} 0.5952..$$

$$C = 36.5^\circ$$

$$15. \frac{4\pi r^3}{3} = \frac{\pi r^2 h}{3} \quad \text{Vol sphere} = \frac{4\pi r^3}{3}$$

$$\text{So } 4r = h \quad \text{Vol cone} = \frac{1}{3} \pi r^2 h$$

16.

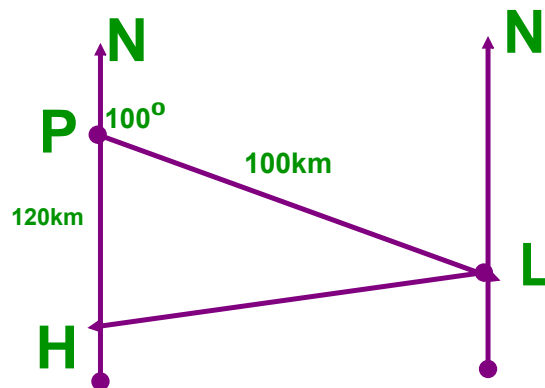
- a) $\sin x = 0.3$
- b) $2\sin x = 0.3$
- c) $\sin 2x = 0.3$
- d) $\sin(x+20) = 0.3$
- e) $2 + \sin x = 1.3$

- a) $x = 17.5, 162.5$
- b) $\sin x = 0.15$ so $x = 8.6, 171.4$
- c) $\sin(17.5) = 0.3$
so $x = 17.5/2 = 8.75$
or $x = 162.5/2 = 81.25$
or $x = 377.5/2 = 188.75$
or $x = 522.5/2 = 261.25$
- d) $\sin(17.5) = 0.3$ but x would be negative;
 $\sin(162.5) = 0.3$
so $\sin(142.5+20) = 0.3$, $x = 142.5$
 $\sin(377.5) = 0.3$
so $\sin(357.5+20) = 0.3$, $x = 357.5$
- e) $2 + \sin x = 1.3$
 $\sin x = -0.7$
 $x = 224.4, x = 315.6$

A ship leaves a port P, and sails 100km on a bearing of 100° to an island, L. It then travels to a harbour H, 120km due south of P.

a) On what bearing is the island from the Port? $100 + 180 = 280\text{km}$ (using alternate angles)

b) Find the distance of the Island from the harbour.



Angle $HPL = 80^\circ$ (Angles on a line total 180°)

Using the cosine rule:

$$HL^2 = 120^2 + 100^2 - 2 \times 100 \times 120 \times \cos 80$$

$$HL^2 = 20232$$

$$HL = 142.2\text{km}$$